TRACCIA 20/6/12

ESERCIZIO N. 2

$X, Y \sim N(1,1) \qquad E[XY] = 2$

$E[2X - (X+Y)^2] = E[2X - (X^2 + Y^2 + 2XY)] =$

$= E[2X - X^2 - Y^2 - 2XY] =$

$= E[2X] + E[-X^2] + E[-Y^2] + E[-2XY] =$

$= 2E[X] - E[X^2] - E[Y^2] - 2E[XY] =$

$Var(X) = E[X^2] - E[X]^2 \Rightarrow E[X^2] = Var(X) + E[X]^2$

$E[X^2] = 1 + (1)^2 = 2$

$E[Y^2] = Var(Y) + E[Y]^2 = 1 + (1)^2 = 2 \Rightarrow$

$= 2(1) - 2 - 2 - 2(2) = 2 - 2 - 2 - 4 = -6$

($2 \leftarrow E[X \cdot Y]$)

$Var(5 + 2X - Y) = Var(5) + Var(2X - Y) = 0 + Var(2X - Y)$

$Var(2X - Y) = Var(2X) + Var(-Y) + 2(E[2X \cdot (-Y)] - E[2X] \cdot E[-Y]) =$

$= 4\,Var(X) + Var(Y) + 2(-2E[XY] - 2E[X] \cdot E[Y]) =$

$= 4(1) + 1 + 2(-2(2) - 2(1 \cdot 1)) = 4 + 1 + 2(-4 - 2) =$

($2 \leftarrow E[XY]$)

$= 5 + 2(-6) = 5 - 12 = -7$